Monopole determinant in Yang–Mills theory

at finite temperature

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Abstract

The fluctuation determinant in the BPS monopole background is calculated in the finite–temperature $SU(2)$ gauge theory.

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1 Introduction

It is well known that asymptotically free gauge theories become weakly coupled at high temperature because of the decrease of the running coupling constant $g^2(T)$ with T. But due to infrared divergences the domain of applicability of perturbation theory is limited by the few lowest orders and by distances much smaller than $(g^2T)^{-1}$, beyond which nonperturbative effects become important. On the other hand, the smallness of the effective coupling justify an application of semiclassical methods, which provide essentially nonperturbative information. While the finite temperature instanton calculations have been performed in detail [1], the contribution of another type of finite action classical solutions, the BPS monopoles [2], was studied only numerically [3]. Analogously to the 't Hooft–Polyakov monopoles in three dimensional adjoint Higgs model, which generate a mass gap and an area law for Wilson loops [4], the BPS monopoles contribute, in principle, to the spatial string tension and to the magnetic screening, responsible for the infrared divergences of perturbation theory.

The mean density of the BPS monopoles is determined in a dilute gas approximation by the contribution of a single monopole to the partition function. The calculation of the latter implies an account of the fluctuations in the monopole background. The purpose of the present paper is to calculate the corresponding fluctuation determinant. The method we use is based on the fact that the propagators in the monopole background are known explicitly [5]–[7]. This method allows to find completely the functional dependence of the determinant on the monopole size. However, the general expressions are too cumbersome, and only the results for the monopoles of a small size will be exhibited explicitly.

It is not evident that the semiclassical calculations for the BPS monopoles can at all be performed. The point is that the Coulomb–like behavior of the monopole field may lead to the logarithmic infrared divergences at a one–loop level. Such divergences indeed have been found in a similar problem of calculation of the quantum corrections to the mass of the 't Hooft–Polyakov monopole [8]. The observation that the BPS monopole can be obtained as an infinite size limit of the periodic instanton [6, 1] does not help, since the instanton determinant is singular in this limit. However, this singularity is a consequence of the fact that the leading contribution to the determinant comes from the distances much larger than the instanton size [1], where the field of the instanton differs considerably from that of the monopole. The monopole determinant, as it will be shown, is finite and free from infrared divergences.

2 Monopole contribution to partition function

In a standard formulation of a field theory at finite temperature the partition function and correlators are represented in a form of Euclidean functional integrals with periodic boundary conditions in an imaginary time with a period, equal to the inverse temperature $\beta = 1/T$. The static fields are obviously periodic and thus contribute to the functional

integral. We consider pure Yang–Mills theory with gauge group $SU(2)$. It possesses (anti)self–dual static classical solutions of a monopole type [2]:

$$A_0^a = q\mu n_a \left(\coth \mu r - \frac{1}{\mu r} \right),$$

$$A_i^a = \varepsilon_{aij} n_j \frac{1}{r} \left(1 - \frac{\mu r}{\sinh \mu r} \right),$$

$$F_{i0}^a = q n_a n_i \frac{1}{r^2} \left[1 - \frac{(\mu r)^2}{\sinh^2 \mu r} \right] + q\mu^2 \left(\delta_{ai} - n_a n_i \right) \frac{1}{\sinh \mu r} \left(\coth \mu r - \frac{1}{\mu r} \right),$$

$$F_{ij}^a = -q\varepsilon_{ijk} F_{k0}^a, \tag{2.1}$$

where $n_i = x_i/r$, $r = |\mathbf{x}|$ and $q = \pm 1$ is a magnetic charge of the solution; in what follows the monpoles with $q = 1$ are considered. We shall also use a matrix notation for gauge potentials: $A_\mu = A_\mu^a \sigma^a / 2i$.

It is worth mentioning that A_0 in (2.1) does not decrease at infinity. Consideration of a constant background shows [1] that not all such fields, but only that, satisfying the condition

$$\lim_{|\mathbf{x}| \to \infty} \mathrm{P} \exp \int_0^\beta dx_0 \, A_0(x_0, \mathbf{x}) = \pm \mathbf{1}, \tag{2.2}$$

give finite contribution to the partition function in the thermodynamic limit, others being suppressed by a factor $\exp(-cVT^3)$. The condition (2.2) leads to a quantization of the monopole size:

$$\mu = \frac{2\pi k}{\beta}. \tag{2.3}$$

The integer k is nothing but the topological charge of a monopole:

$$k = \frac{q}{32\pi^2} \int_0^\beta dx_0 \int d^3x \, \frac{1}{2} \varepsilon_{\mu\nu\lambda\rho} F_{\mu\nu}^a F_{\lambda\rho}^a.$$

Only the solutions with μ, given by (2.3), can be gauge transformed to the periodic fields decreasing at infinity. This gauge transformation is periodic for even and antiperiodic for odd k. When the matter fields are added, only the periodic gauge transformations are allowed, so in this case the topological charge of a monopole should be even. The possibility of antiperiodic gauge transformations in pure gauge theory is a consequence of $\mathbf{Z}_2$ symmetry [9]. However, at high temperature $\mathbf{Z}_2$ symmetry is spontaneously broken, thus the contribution of the monopoles with odd topological charge apparently should not be taken into account in this case as well.

The monopole density can be calculated by standard semiclassical techniques. The zero modes are treated by collective coordinate procedure. Since the BPS monopole is self–dual, the contribution of non–zero modes can be expressed through the spin-0 fluctuation determinant [10]. In a background gauge $D_\mu^{(\mathrm{cl})} A_\mu = 0$ it gives, together with a ghost contribution, a factor $[\det(-D^2)]^{-1}$, where the covariant derivative acts in the adjoint

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representation: $D_\mu^{ab} = \delta^{ab}\partial_\mu + \varepsilon^{acb}A_\mu^c$. Thus one obtains for a one–monopole partition function:

$$Z_1 = \sum_k \int d\nu \, J^{1/2} \left[\det\left(-D^2\right)\right]^{-1} e^{-8\pi^2 k/g_0^2}, \tag{2.4}$$

where the integration is over collective coordinates, corresponding to the zero modes. The Jacobian J ensures a correct normalization of the measure and depends on a parametrization of the classical solutions. The last factor is a contribution of the classical action.

There are four evident zero modes in the monopole background. Three of them correspond to the global translations and one – to the global gauge transformation, commuting with A_0. Let us stress that there is no zero mode, associated with dilatations – the integration over μ is replaced by summation over k. However, the total number of the zero modes is larger, it grows linearly with k, so the BPS monopole (2.1) is a particular case of a more general periodic solution with a unit magnetic charge and topological charge k. Such solutions are generally nonstatic and, in principle, all of them can be classified [11]. More detailed treatment of the zero mode contribution is beyond the scope of the present paper.

The monopole density is ultraviolet divergent due to the contribution of both fluctuations and zero modes. After a proper regularization all ultraviolet infinities are absorbed by a standard one–loop coupling constant renormalization, and the divergent contribution reduces to the replacement of the bare coupling g_0^2 in (2.4) by the renormalized one $g^2(T)$.

For the monopoles with nonintegral topological charge the fluctuation determinant contains also the infrared divergences. We regularize them by cutting divergent integrals on an upper bound. The leading divergence is proportional to the volume, it produce the factor $\exp(-cVT^3)$, which leads to the abovementioned quantization of a monopole size. This divergence, as well as the nonleading power–like and logarithmic ones, cancel, when the condition (2.3) is satisfied.

We shall perform explicit calculations of the determinant for large k, systematically dropping $O(1/k)$ terms. Here we quote the final result:

$$-\ln\det\left(-D^2\right) = \frac{2}{3}k\ln\frac{T}{\Lambda} + \frac{2}{3}k\ln k - 0.240373k + O\left(k^0\right), \tag{2.5}$$

where Λ is ultraviolet cutoff.

3 Variation of determinant

We use the following representation for the regularized determinant:

$$\Gamma \equiv -\ln\det\left(-D^2\right) = \int_{1/\Lambda^2}^\infty \frac{dt}{t} \, \mathrm{Sp} \, e^{tD^2}. \tag{3.1}$$

The variation of Γ can be expressed through the Green function of $(-D^2)$. We choose μ to be a parameter of variation, then

$$\mu\frac{\partial A_\nu}{\partial\mu} = A_\nu + x_j\partial_j A_\nu = x_j F_{j\nu} + D_\nu(x_j A_j) + \delta_{\nu 0}A_0 = x_j F_{j\nu} + \delta_{\nu 0}A_0 \qquad (3.2)$$

and

$$\mu\frac{\partial\Gamma}{\partial\mu} = \frac{1}{2}\int_{1/\Lambda^2}^{\infty}\frac{dt}{t}\int_0^t ds\ \text{Sp}\left(e^{sD^2}\mu\frac{\partial D^2}{\partial\mu}e^{(t-s)D^2} + e^{(t-s)D^2}\mu\frac{\partial D^2}{\partial\mu}e^{sD^2}\right). \qquad (3.3)$$

Consider the integrand in this expression:

$$\mathcal{F}(t,s) = \text{Sp}\left(e^{sD^2}\mu\frac{\partial D^2}{\partial\mu}e^{(t-s)D^2} + e^{(t-s)D^2}\mu\frac{\partial D^2}{\partial\mu}e^{sD^2}\right). \qquad (3.4)$$

At first sight it does not depend on s due to the cyclic property of the trace, equivalent for differential operators to integration by parts. This, however, leaves boundary terms and, as a result, $\mathcal{F}$ is a second order polynomial in s. To show it, let us differentiate (3.4) n times with respect to s:

$$\frac{\partial^n}{\partial s^n}\mathcal{F}(t,0) = \text{Sp}\left(\mathcal{O}_{n+1}e^{tD^2} + (-1)^n e^{tD^2}\mathcal{O}_{n+1}\right). \qquad (3.5)$$

Here $\mathcal{O}_n$ is a differential operator of n–th order, defined recursively by

$$\mathcal{O}_1 = \mu\frac{\partial D^2}{\partial\mu} = \left\{\mu\frac{\partial A_\nu}{\partial\mu}, D_\nu\right\}, \qquad (3.6)$$

$$\mathcal{O}_{n+1} = \left[D^2, \mathcal{O}_n\right]. \qquad (3.7)$$

It is not difficult to write down explicitly the first few ones:

$$\mathcal{O}_2 = \left\{D_\nu, \left\{\mu\frac{\partial A_\lambda}{\partial\mu}, F_{\nu\lambda}\right\}\right\} + \left\{D_\nu, \left\{\left(D_\nu\mu\frac{\partial A_\lambda}{\partial\mu}\right), D_\lambda\right\}\right\}, \qquad (3.8)$$

$$
\begin{aligned}
\mathcal{O}_3 = {} & \left\{D_\lambda, \left\{D_\nu, \left\{\mu\frac{\partial A_\rho}{\partial\mu}, (D_\lambda F_{\nu\rho})\right\}\right\}\right\} + \left\{F_{\lambda\nu}, \left\{\mu\frac{\partial A_\rho}{\partial\mu}, F_{\nu\rho}\right\}\right\} \\
& + \left\{D_\nu, \left\{\left(D_\lambda\mu\frac{\partial A_\rho}{\partial\mu}\right), F_{\nu\rho}\right\}\right\} + \left\{F_{\lambda\nu}, \left\{\left(D_\nu\mu\frac{\partial A_\rho}{\partial\mu}\right), D_\rho\right\}\right\} \\
& + \left\{D_\nu, \left\{\left(D_\lambda D_\nu\mu\frac{\partial A_\rho}{\partial\mu}\right), D_\rho\right\}\right\} + \left\{D_\nu, \left\{\left(D_\nu\mu\frac{\partial A_\rho}{\partial\mu}\right), F_{\lambda\rho}\right\}\right\}. \qquad (3.9)
\end{aligned}
$$

It follows from the definition of $\mathcal{O}_{n+1}$ that for $n \geq 1$ the right hand side of eq. (3.5) is given by the trace of a commutator or, equivalently, by an integral of a total derivative, which reduces to a surface term:

$$\frac{\partial^n}{\partial s^n}\mathcal{F}(t,0) = \text{Sp}\left[D^2, \mathcal{O}_n e^{tD^2} + (-1)^n e^{tD^2}\mathcal{O}_n\right]$$

$$
\begin{aligned}
= \int_0^\beta dx_0 \int d^3x \, \mathrm{Tr} \, \Big[&\overrightarrow{D}^2 \left(\overrightarrow{\mathcal{O}}_n \mathcal{K}(x,y;t) + (-1)^n \mathcal{K}(x,y;t) \overleftarrow{\mathcal{O}}_n \right) \\
& - \left(\overrightarrow{\mathcal{O}}_n \mathcal{K}(x,y;t) + (-1)^n \mathcal{K}(x,y;t) \overleftarrow{\mathcal{O}}_n \right) \overleftarrow{D}^2 \Big]\Big|_{y=x}
\end{aligned}
$$

$$
\begin{aligned}
= \int_0^\beta dx_0 \oint_{S_R} d^2\Sigma \, n_i \, \mathrm{Tr} \, \Big[&\overrightarrow{D}_i \left(\overrightarrow{\mathcal{O}}_n \mathcal{K}(x,y;t) + (-1)^n \mathcal{K}(x,y;t) \overleftarrow{\mathcal{O}}_n \right) \\
& + \left(\overrightarrow{\mathcal{O}}_n \mathcal{K}(x,y;t) + (-1)^n \mathcal{K}(x,y;t) \overleftarrow{\mathcal{O}}_n \right) \overleftarrow{D}_i \Big]\Big|_{y=x}, \quad (3.10)
\end{aligned}
$$

where Tr denotes the trace over color indices in the adjoint representation and $R \to \infty$. By $\mathcal{K}$ we denote a heat kernel of $-D^2$:

$$
\mathcal{K}(x,y;t) = \left\langle x \, \middle| \, \mathrm{e}^{tD^2} \, \middle| \, y \right\rangle. \tag{3.11}
$$

By noting that $D_\nu \mu \frac{\partial A_\lambda}{\partial \mu}$ falls exponentially at infinity and $F_{\nu\lambda}$ is of order $1/r^2$ we conclude that the coefficients of the operator $\mathcal{O}_3$, and thus of all $\mathcal{O}_n$ with $n \geq 3$, are, at least, of order $1/r^3$. As a consequence, the surface integral (3.10) vanishes for $n \geq 3$ and only the first and the second derivatives of $\mathcal{F}$ are not equal to zero.

Using an explicit expressions for $\mathcal{O}_1$ and $\mathcal{O}_2$ and taking into account that only the variation of A_0 does not decrease at infinity we find:

$$
\begin{aligned}
\frac{\partial}{\partial s} \mathcal{F}(t,0) = 2 \int_0^\beta dx_0 \oint_{S_R} d^2\Sigma \, n_i \, \mathrm{Tr} \, \Big[&\mu \frac{\partial A_0}{\partial \mu} \Big(\overrightarrow{D}_i \overrightarrow{D}_0 \mathcal{K}(x,y;t) \\
& - \overrightarrow{D}_i \mathcal{K}(x,y;t) \overleftarrow{D}_0 + \overrightarrow{D}_0 \mathcal{K}(x,y;t) \overleftarrow{D}_i - \mathcal{K}(x,y;t) \overleftarrow{D}_0 \overleftarrow{D}_i \Big)\Big|_{y=x} \Big]
\end{aligned}
$$

$$
= 4 \int_0^\beta dx_0 \oint_{S_R} d^2\Sigma \, n_i \, \mathrm{Tr} \, \left(\mu \frac{\partial A_0}{\partial \mu} F_{i0} \mathcal{K}(x,x;t) \right), \tag{3.12}
$$

$$
\begin{aligned}
\frac{\partial^2}{\partial s^2} \mathcal{F}(t,0) = 4 \int_0^\beta dx_0 \oint_{S_R} d^2\Sigma \, n_i \, \mathrm{Tr} \, \Big[&\mu \frac{\partial A_0}{\partial \mu} F_{j0} \Big(\overrightarrow{D}_i \overrightarrow{D}_j \mathcal{K}(x,y;t) \\
& + \overrightarrow{D}_i \mathcal{K}(x,y;t) \overleftarrow{D}_j + \overrightarrow{D}_j \mathcal{K}(x,y;t) \overleftarrow{D}_i + \mathcal{K}(x,y;t) \overleftarrow{D}_j \overleftarrow{D}_i \Big)\Big|_{y=x} \Big]
\end{aligned}
$$

$$
= -\frac{8}{t} \int_0^\beta dx_0 \oint_{S_R} d^2\Sigma \, n_i \, \mathrm{Tr} \, \left(\mu \frac{\partial A_0}{\partial \mu} F_{i0} \mathcal{K}(x,x;t) \right). \tag{3.13}
$$

The last equality holds, because at infinity

$$
\overrightarrow{D}_i \overrightarrow{D}_j \mathcal{K}(x,y;t) = \left[-\frac{1}{2t}\delta_{ij} + \frac{1}{4t^2}(x-y)_i(x-y)_j \right] \mathcal{K}(x,y;t) + O\left(\frac{1}{r}\right) \tag{3.14}
$$

and analogous formulas are valid for $\overrightarrow{D}_i \mathcal{K}(x,y;t) \overleftarrow{D}_j$ and $\mathcal{K}(x,y;t) \overleftarrow{D}_j \overleftarrow{D}_i$. They can be derived directly from the defining equation for the heat kernel with the use of it's asymptotic form at $t \to 0$:

$$
\mathcal{K}^{ab}(x,y;t) \sim \frac{\delta^{ab}}{16\pi^2 t^2} \, \mathrm{e}^{-(x-y)^2/4t}. \tag{3.15}
$$

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Thus we obtain:

$$
\begin{aligned}
\mathcal{F}(t,s) &= 2\,\mathrm{Sp}\left(\mu\frac{\partial D^2}{\partial\mu}\,\mathrm{e}^{tD^2}\right) \\
&\quad +4\left(s-\frac{s^2}{t}\right)\int_0^\beta dx_0\oint_{S_R} d^2\Sigma\,n_i\,\mathrm{Tr}\left(\mu\frac{\partial A_0}{\partial\mu}F_{i0}\mathcal{K}(x,x;t)\right).
\end{aligned}
\tag{3.16}
$$

Substituting this expression in the variation of the determinant, eq. (3.3), and doing the integral over s one finds:

$$
\begin{aligned}
\mu\frac{\partial\Gamma}{\partial\mu} &= \int_{1/\Lambda^2}^\infty dt\,\mathrm{Sp}\left(\mu\frac{\partial D^2}{\partial\mu}\,\mathrm{e}^{tD^2}\right) \\
&\quad +\frac{1}{3}\int_{1/\Lambda^2}^\infty dt\,t\int_0^\beta dx_0\oint_{S_R} d^2\Sigma\,n_i\,\mathrm{Tr}\left(\mu\frac{\partial A_0}{\partial\mu}F_{i0}\mathcal{K}(x,x;t)\right).
\end{aligned}
\tag{3.17}
$$

After the change of variables $t\to t-1/\Lambda^2$ the integration over t can be easily performed. As it was shown in ref. [12], the first term in (3.17) remains finite in $\Lambda\to\infty$ limit and is given by

$$
\mu\frac{\partial\Gamma_I}{\partial\mu}=\int_0^\beta dx_0\int d^3x\,\varepsilon^{acb}\mu\frac{\partial A_\nu^c}{\partial\mu}\mathcal{J}_\nu^{ba},
\tag{3.18}
$$

$$
\mathcal{J}_\nu^{ba}(x)=\left(\overrightarrow{D}_\nu^{bd}\,\mathcal{G}_R^{da}(x,y)+\mathcal{G}_R^{bd}(y,x)\overleftarrow{D}_\nu^{da}\right)\Bigg|_{y=x},
\tag{3.19}
$$

where $\mathcal{G}_R$ is the Green function, $\mathcal{G}=(-D^2)^{-1}$, regularized by the point splitting:

$$
\mathcal{G}_R^{ab}(x,y)=\mathcal{G}^{ab}(x,y)-\frac{\frac{1}{2}\mathrm{tr}\,\sigma^a\Phi(x,y)\sigma^b\Phi(y,x)}{4\pi^2(x-y)^2},
\tag{3.20}
$$

$$
\Phi(x,y)=\mathrm{P}\exp\int_x^y dx'_\mu\,A_\mu(x').
\tag{3.21}
$$

The second term in (3.17) reduces to a sum of two surface integrals:

$$
\mu\frac{\partial\Gamma_{II}}{\partial\mu}=\frac{1}{3}\int_0^\beta dx_0\oint_{S_R}d^2\Sigma\,n_i\,\mathrm{Tr}\left[\mu\frac{\partial A_0}{\partial\mu}F_{i0}\left\langle x\left|(-D^2)^{-2}\,\mathrm{e}^{D^2/\Lambda^2}\right|x\right\rangle\right]
\tag{3.22}
$$

and

$$
\mu\frac{\partial\Gamma_{III}}{\partial\mu}=\frac{1}{3}\int_0^\beta dx_0\oint_{S_R}d^2\Sigma\,n_i\,\mathrm{Tr}\left[\mu\frac{\partial A_0}{\partial\mu}F_{i0}\frac{1}{\Lambda^2}\left\langle x\left|(-D^2)^{-1}\,\mathrm{e}^{D^2/\Lambda^2}\right|x\right\rangle\right].
\tag{3.23}
$$

In the last expression $(-D^2)^{-1}$ can be replaced by a free propagator and the asymptotic form (3.15) of the heat kernel can be used. One gets in the $\Lambda\to\infty$ limit

$$
\frac{1}{\Lambda^2}\left\langle x\left|(-D^2)^{-1}\,\mathrm{e}^{D^2/\Lambda^2}\right|x\right\rangle\sim\frac{1}{\Lambda^2}\int d^4y\,\frac{1}{4\pi^2(x-y)^2}\frac{\Lambda^4}{16\pi^2}\,\mathrm{e}^{-(y-x)^2\Lambda^2/4}=\frac{1}{16\pi^2},
$$

and after simple transformations, involving the use of the equations of motion, (3.23) can be rewritten as

$$
\mu\frac{\partial\Gamma_{III}}{\partial\mu}=\frac{1}{3}\frac{1}{64\pi^2}\mu\frac{\partial}{\partial\mu}\int_0^\beta dx_0\int d^3x\,\mathrm{Tr}\,F_{\nu\lambda}F_{\nu\lambda}=-\frac{1}{3}\mu\frac{\partial k}{\partial\mu}.
$$

Thus

$$\Gamma_{III} = -\frac{1}{3}k. \tag{3.24}$$

Eqs. (3.18), (3.19) and (3.22) express the variation of Γ_I and Γ_{II} in terms of the isospin–1 propagator in the monopole background. It is convenient to use the ADHM construction for self–dual gauge fields [13], developed for the BPS monopoles in ref. [7]; which provides the expressions for propagators in a rather compact form [14, 15]. It is worth mentioning that the ADHM construction enables to calculate the multi–instanton determinants in complete generality [12, 16].

4   Ultraviolet finite part of determinant

The essence of the ADHM construction is introduction of an auxiliary linear space; that of 2×2 matrix–valued functions of a new variable $z \in [-1/2, 1/2]$ in the case of $SU(2)$ monopoles. The scalar product in this space is defined by

$$\langle v_1 | v_2 \rangle = \int_{-1/2}^{1/2} dz \, v_1^\dagger(z) v_2(z), \tag{4.1}$$

where matrix multiplication is implied. Thus $\langle v_1 | v_2 \rangle$ is a 2×2 matrix. The one–monopole solution is given in these terms by

$$A_\mu(x) = \langle v(x) | \partial_\mu v(x) \rangle, \tag{4.2}$$

where [7]

$$|v(x)\rangle = \left(\frac{\xi}{\sinh \xi} \right)^{1/2} \exp\left(i\xi_0 z - \xi_i \sigma^i z \right) \tag{4.3}$$

is a solution to the equation

$$\Delta^\dagger(x) |v(x)\rangle \equiv \left(i\frac{d}{dz} + \xi_0 + i\xi_i\sigma^i \right) |v(x)\rangle = 0, \tag{4.4}$$

normalized by a condition

$$\langle v(x) | v(x) \rangle = \mathbf{1}. \tag{4.5}$$

Here and in what follows we denote by ξ_μ the rescaled coordinate:

$$\xi_\nu = \mu x_\nu, \quad \xi \equiv |\boldsymbol{\xi}|. \tag{4.6}$$

An important property of the ADHM construction is that operator $\Delta^\dagger(x)\Delta(x)$ is scalar, i.e. proportional to the unit matrix, and positive definite. We shall need an explicit expression for it's Green function [7]:

$$
\begin{aligned}
f(x; z, z') &\equiv \langle z | (\Delta^\dagger(x)\Delta(x))^{-1} | z' \rangle \\
&= -\frac{1}{2\xi} e^{i\xi_0(z-z')} \left(\sinh\xi |z - z'| + \coth\frac{\xi}{2} \sinh\xi z \sinh\xi z' \right. \\
&\qquad \left. - \tanh\frac{\xi}{2} \cosh\xi z \cosh\xi z' \right).
\end{aligned}
\tag{4.7}
$$

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The isospin–1 Green function of $(-D^2)$ is expressed through $|v\rangle$ in a relatively simple way [15]:

$$
\begin{aligned}
G^{ab}(x,y) \;=\;& \frac{\frac{1}{2}\operatorname{tr}\sigma^a\langle v(x)|v(y)\rangle\sigma^b\langle v(y)|v(x)\rangle}{4\pi^2(x-y)^2} \\
&+\frac{1}{4\pi^2}\int_{-1/2}^{1/2} dz_1\,dz_2\,dz_3\,dz_4\, M(z_1,z_2,z_3,z_4) \\
&\times\frac{1}{2}\operatorname{tr}\left(v^\dagger(x,z_1)v(x,z_2)\sigma^a\right)\operatorname{tr}\left(v^\dagger(y,z_4)v(y,z_3)\sigma^b\right),
\end{aligned}
\tag{4.8}
$$

where [7]

$$
\begin{aligned}
M(z_1,z_2,z_3,z_4) \;=\;& -\frac{\mu^2}{4}\delta(z_1-z_2-z_3+z_4)\left[\,|z_1+z_2-z_3-z_4|-1\right. \\
&+\left.|z_1-z_2|+\frac{(z_1+z_2)(z_3+z_4)}{1-|z_1-z_2|}\right].
\end{aligned}
\tag{4.9}
$$

This Green function does not obey a periodicity condition. The periodic one is obtained from it by standard procedure:

$$
\mathcal{G}^{ab}(x,y) = \sum_{n=-\infty}^{+\infty} G^{ab}(x_0,\mathbf{x};y_0+n\beta,\mathbf{y}).
\tag{4.10}
$$

It is convenient to represent regularized Green function (3.20) as a sum of three terms:

$$
\mathcal{G}_R^{ab}(x,y) = \mathcal{G}^{(1)ab}(x,y) + \mathcal{G}^{(2)ab}(x,y) + \mathcal{G}^{(3)ab}(x,y),
\tag{4.11}
$$

where

$$
\begin{aligned}
\mathcal{G}^{(1)ab}(x,y) \;=\;& \frac{\frac{1}{2}\operatorname{tr}\left(\sigma^a\langle v(x)|v(y)\rangle\sigma^b\langle v(y)|v(x)\rangle-\sigma^a\Phi(x,y)\sigma^b\Phi(y,x)\right)}{4\pi^2(x-y)^2} \\
=\;& \frac{1}{4\pi^2}\frac{1}{2}\operatorname{tr}\left(\sigma^a R(x,y)\sigma^b\Phi(y,x)+\sigma^a\Phi(x,y)\sigma^b R(y,x)\right) \\
&+O\left((x-y)^2\right),
\end{aligned}
\tag{4.12}
$$

$$
R(x,y) = \frac{\langle v(x)|v(y)\rangle - \Phi(x,y)}{(x-y)^2},
\tag{4.13}
$$

$$
\mathcal{G}^{(2)ab}(x,y) = \sum_{n\neq 0}\frac{\frac{1}{2}\operatorname{tr}\sigma^a\left\langle v(x)\left|e^{2\pi i knz}\right|v(y)\right\rangle\sigma^b\left\langle v(y)\left|e^{-2\pi i knz}\right|v(x)\right\rangle}{4\pi^2\left[(x_0-y_0-n\beta)^2+(\mathbf{x}-\mathbf{y})^2\right]}.
\tag{4.14}
$$

In the last expression it was taken into account that

$$
|v(y_0+n\beta,\mathbf{y})\rangle = e^{2\pi i knz}\,|v(y_0,\mathbf{y})\rangle,
\tag{4.15}
$$

where k is defined by (2.3). Here and in what follows we consider k as a continuous variable, and put it to be integer only at the end of the calculation. Using eq. (4.15) one

can transform $\mathcal{G}^{(3)}$, which comes from the second term in (4.8), into a more simple form. Really, summation over n in (4.10) gives rise to a factor

$$\sum_n \mathrm{e}^{2\pi i k n(z_3 - z_4)} = \frac{1}{k} \sum_n \delta\left(z_3 - z_4 - \frac{n}{k}\right), \tag{4.16}$$

which together with the δ–function in the kernel (4.9) allows to eliminate an integration over z_2 and z_3 in eq. (4.8). Changing variables in the remaining integrals to $z = 2z_1 - n/k$ and $w = 2z_4 + n/k$ one finally gets:

$$
\begin{aligned}
\mathcal{G}^{(3)ab}(x,y) &= -\frac{\mu^2}{16\pi^2 k} \sum_{n=-N}^{N} \frac{1}{4} \int_{-1+|n|/k}^{1-|n|/k} dz \, dw \left(|z-w| - 1 + \frac{|n|}{k} + \frac{zw}{1 - \frac{|n|}{k}}\right) \\
&\quad \times \frac{1}{2} \operatorname{tr}\left(u_{-n}(x,z)\sigma^a\right) \operatorname{tr}\left(u_n(y,w)\sigma^b\right),
\end{aligned}
\tag{4.17}
$$

where

$$u_n(x,z) = \frac{\xi}{\sinh \xi} \exp\left(i\xi_0 \frac{n}{k} - \xi_i \sigma^i z\right) \tag{4.18}$$

and N is an integral part of k. In (4.17) n varies from $-N$ to N, because in (4.16) $|z_3 - z_4|$ is by definition smaller than unity.

The next step is evaluation of the current $\mathcal{J}_\mu$, but first we rescale integration variables in eq. (3.18) and express the trace in the adjoint representation in it through the usual matrix trace:

$$k\frac{\partial \Gamma_I}{\partial k} = \int_0^{2\pi k} d\xi_0 \int d^3\xi \, \operatorname{tr}\left(\mu \frac{\partial A_\nu}{\partial \mu} J_\nu\right) = 8\pi^2 k \int_0^{\lambda k} d\xi \, \xi^2 \operatorname{tr}\left[(\xi_j F_{j\nu} + \delta_{\nu 0} A_0) J_\nu\right], \tag{4.19}$$

where $\lambda = 2\pi R/\beta$, R is an infrared cutoff. The last equality holds, because an integrand depends only on $|\boldsymbol{\xi}|$ in virtue of the central symmetry and time independence of the monopole solution.

Consider first the contribution of $\mathcal{G}^{(1)}$. It can be rewritten in the form (4.19) with the help of the identities

$$
\begin{aligned}
\overrightarrow{D}_\mu^{ad} \frac{1}{2} \operatorname{tr}\left(\sigma^d A \sigma^b B\right) &= \frac{1}{2} \operatorname{tr}\left[\sigma^a(\overrightarrow{D}_\mu A)\sigma^b B - \sigma^a A \sigma^b(B\overleftarrow{D}_\mu)\right], \\
\frac{1}{2} \operatorname{tr}\left(\sigma^a A \sigma^d B\right)\overleftarrow{D}_\mu^{db} &= \frac{1}{2} \operatorname{tr}\left[\sigma^a(A\overleftarrow{D}_\mu)\sigma^b B - \sigma^a A \sigma^b(\overrightarrow{D}_\mu B)\right],
\end{aligned}
\tag{4.20}
$$

where the covariant derivative on the right hand side acts in the fundamental representation. Applying them to (4.12) and taking into account that

$$\overrightarrow{D}_\mu \Phi(x,y)\Big|_{y=x} = 0 = \Phi(y,x)\overleftarrow{D}_\mu\Big|_{y=x},$$

one finds that $\mathcal{J}_\mu^{(1)ba} = \frac{1}{4} i\varepsilon^{abd} \operatorname{tr}\left(\sigma^d J_\mu^{(1)}\right)$ with

$$J_\mu^{(1)}(x) = \frac{1}{\pi^2}\left(\overrightarrow{D}_\mu R(x,y) + R(y,x)\overleftarrow{D}_\mu\right)\Big|_{y=x}. \tag{4.21}$$

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Now one may use the ADHM construction to write [12]:

$$J_\mu^{(1)} = \frac{1}{3\pi^2} \left\langle v \left| f \left(e_\mu \Delta^\dagger - \Delta e_\mu^\dagger \right) f \right| v \right\rangle, \qquad e_\mu = (1, -i\boldsymbol{\sigma}). \tag{4.22}$$

From (4.3) and (4.7) we find

$$f|v\rangle = -\frac{1}{2} (\xi \sinh \xi)^{-1/2} \, e^{\,i\xi_0 z} (a - n_i \sigma^i b), \tag{4.23}$$

$$a = z \sinh \xi z - \frac{1}{2} \tanh \frac{\xi}{2} \cosh \xi z, \tag{4.24}$$

$$b = z \cosh \xi z - \frac{1}{2} \coth \frac{\xi}{2} \sinh \xi z. \tag{4.25}$$

Substituting (4.23) in eq. (4.22) one obtains:

$$
\begin{aligned}
J_0^{(1)} &= \frac{1}{6\pi^2 \sinh \xi} \int_{-1/2}^{1/2} dz \left[i n_i \sigma^i (a^2 + b^2) - 2iab \right] \\
&= i n_i \sigma^i \frac{1}{12\pi^2} \left(\frac{1}{\xi^3} - \frac{\cosh \xi}{\sinh^3 \xi} \right),
\end{aligned}
\tag{4.26}
$$

$$
\begin{aligned}
J_i^{(1)} &= \frac{1}{6\pi^2 \xi \sinh \xi} \int_{-1/2}^{1/2} dz \left\{ i\varepsilon_{ijk} n_j \sigma^k \left[\xi(a^2 - b^2) + b\frac{da}{dz} - a\frac{db}{dz} \right] \right. \\
&\quad \left. + \sigma^i a \frac{da}{dz} + n_j n_k \sigma^j \sigma^i \sigma^k b \frac{db}{dz} - n_i \left(a\frac{db}{dz} + b\frac{da}{dz} \right) \right\} \\
&= i\varepsilon_{ijk} n_j \sigma^k \frac{1}{12\pi^2 \sinh \xi} \left(\frac{1}{\sinh^2 \xi} + \frac{\cosh \xi}{\xi \sinh \xi} - \frac{2}{\xi^2} \right).
\end{aligned}
\tag{4.27}
$$

For the first factor in the integrand in eq. (4.19) we have:

$$\xi_j F_{j0} + A_0 = \frac{\sigma^a n_a}{2i} \left(\coth \xi - \frac{\xi}{\sinh^2 \xi} \right) \tag{4.28}$$

$$\xi_j F_{ji} = \varepsilon_{ija} n_j \frac{\sigma^a}{2i} \frac{\xi}{\sinh \xi} \left(\coth \xi - \frac{1}{\xi} \right). \tag{4.29}$$

Calculating the trace and doing the integral over ξ one finds:

$$k\frac{\Gamma_I^{(1)}}{\partial k} = \left(\frac{2}{3} \ln \frac{\lambda k}{\pi} - \frac{5}{18} + \frac{\pi^2}{54} + \frac{2\gamma}{3} \right) k, \tag{4.30}$$

where γ is the Euler constant. Note that the term containing $J_0^{(1)}$ is logarithmically divergent – this is the origin of $\ln \lambda$ in (4.30).

Let us turn to the contribution of $\mathcal{G}^{(2)}$. To rewrite it in the form (4.19) we use (4.20) and an obvious equality $D_\mu = \langle v(x) | \partial_\mu | v(x) \rangle$, valid for covariant derivative in the fundamental representation. A simple calculation gives:

$$
\begin{aligned}
\mathcal{J}_\mu^{(2)ab} &= \frac{1}{4\pi^2} \sum_{n\neq 0} \frac{1}{2} \operatorname{tr} \left(\delta_{\mu 0} \frac{4}{\beta^3 n^3} \sigma^a \left\langle v \left| e^{\,2\pi i k n z} \right| v \right\rangle \sigma^b \left\langle v \left| e^{-2\pi i k n z} \right| v \right\rangle \right. \\
&\quad + \frac{1}{\beta^2 n^2} \sigma^a \left\langle v \left| [\partial_\mu \hat{P}, \, e^{\,2\pi i k n z}] \right| v \right\rangle \sigma^b \left\langle v \left| e^{-2\pi i k n z} \right| v \right\rangle \\
&\quad \left. + \frac{1}{\beta^2 n^2} \sigma^a \left\langle v \left| e^{\,2\pi i k n z} \right| v \right\rangle \sigma^b \left\langle v \left| [e^{-2\pi i k n z}, \, \partial_\mu \hat{P}] \right| v \right\rangle \right).
\end{aligned}
\tag{4.31}
$$

with $\hat{P} = |v\rangle\langle v|$. Substituting this expression in (3.18) we rewrite it in the form (4.19) with

$$
\begin{aligned}
J_\mu^{(2)} = {} & \frac{1}{4\pi^2} \sum_{n\neq 0} \left[\frac{1}{(2\pi k)^2 n^2} \left(\langle v | [\partial_\mu \hat{P}, \, \mathrm{e}^{2\pi i knz}] | v \rangle \, \mathrm{tr} \, \langle v | \mathrm{e}^{-2\pi i knz} | v \rangle \right. \right. \\
& \left. - \langle v | \mathrm{e}^{-2\pi i knz} | v \rangle \, \mathrm{tr} \, \langle v | [\partial_\mu \hat{P}, \, \mathrm{e}^{2\pi i knz}] | v \rangle + (n \leftrightarrow -n) \right) \\
& \left. + 4\delta_{\mu 0} \frac{1}{(2\pi k)^3 n^3} \left(\langle v | \mathrm{e}^{2\pi i knz} | v \rangle \, \mathrm{tr} \, \langle v | \mathrm{e}^{-2\pi i knz} | v \rangle - (n \leftrightarrow -n) \right) \right]. \quad (4.32)
\end{aligned}
$$

At first sight $J^{(2)}$, being of order $1/k^2$, gives $O(1/k)$ contribution to Γ. However, the situation is more involved due to infrared divergences. The calculations are lengthy and are carried out in Appendix. Here we only quote the results. The leading cubic divergence has the form

$$
\ln \det \left(-D^2 \right) \sim \frac{16\pi^3 R^3}{9\beta^3} s^2 (1-s)^2 = \frac{4\pi^2}{3} VT^3 s^2 (1-s)^2, \quad (4.33)
$$

where $s = k \bmod 1$. Hence we obtain the expected result, that the contribution of the monopoles with nonintegral topological charge is suppressed by a factor $\exp(-cVT^3)$ with coefficient c precisely the same as for the constant background field [1]. For integral k the cubic divergence vanishes, all other ones also cancel, and $\Gamma_I^{(2)}$ is infrared finite and is indeed of order $1/k$.

Now let us proceed with the contribution of $\mathcal{G}^{(3)}$. Direct calculation of $\mathcal{J}^{(3)}$ according to (3.19) and it's substitution into (3.18) leads, after a simple algebra, to the result, which again has a form of (4.19) with

$$
\begin{aligned}
J_\mu^{(3)} = {} & \frac{1}{32\pi^2 k} \sum_{n=-N}^{N} \int_{-1+|n|/k}^{1-|n|/k} dz\, dw \left(|z - w| - 1 + \frac{|n|}{k} + \frac{zw}{1 - \frac{|n|}{k}} \right) \\
& \left\{ \frac{1}{2} [u_{-n}(z), \partial_\mu u_n(w)] - \frac{1}{2} [\partial_\mu u_{-n}(z), u_n(w)] \right. \\
& + u_{-n}(z) \, \mathrm{tr} \, (u_n(w) A_\mu) + u_n(w) \, \mathrm{tr} \, (u_{-n}(z) A_\mu) \\
& \left. - 2A_\mu \, \mathrm{tr} \, (u_{-n}(z) u_n(w)) + A_\mu \, \mathrm{tr} \, u_{-n}(z) \, \mathrm{tr} \, u_n(w) \right\}. \quad (4.34)
\end{aligned}
$$

Note that the term under consideration do not contain infrared divergences, because $J_0^{(3)} = 0$. It can be shown by an explicit calculation according to eq. (4.34), but more simple way to see it is to use directly a definition (3.19) and eq. (4.17). Really, since $A_0^c \propto n^c$ and $\mathrm{tr} \left(u_n(x, z) \sigma^d \right) \propto n^d$, $\varepsilon^{acb} A_0^c \, \mathrm{tr} \left(u_n(x, z) \sigma^d \right) = 0$ and D_0^{ad} acts on $\mathcal{G}^{(3)db}$ as an ordinary time derivative. Hence $\mathcal{J}_0^{ab} \propto \mathrm{tr} \, (u_{-n} \sigma^a) \, \mathrm{tr} \, (u_n \sigma^b) \propto n^a n^b$, which gives zero after the substitution in eq. (3.18).

The straightforward calculation of $J_i^{(3)}$ gives:

$$
\begin{aligned}
J_i^{(3)} = {} & -i\varepsilon_{ijk} n_j \sigma^k \frac{1}{16\pi^2 k} \frac{\xi^2}{\sinh^3 \xi} \sum_{n=-N}^{N} \int_{-1+|n|/k}^{1-|n|/k} dz\, dw \left(|z - w| - 1 + \frac{|n|}{k} \right. \\
& \left. + \frac{zw}{1 - \frac{|n|}{k}} \right) \sinh \xi z \sinh \xi w
\end{aligned}
$$

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$$= i\varepsilon_{ijk}n_j\sigma^k \frac{1}{16\pi^2 k}\frac{\xi^2}{\sinh^3\xi}\sum_{n=-N}^{N}\left[\frac{2}{\xi^2}\left(1-\frac{|n|}{k}\right)+\frac{1}{\xi^3}\sinh 2\left(1-\frac{|n|}{k}\right)\xi\right.$$

$$\left.-\frac{2}{\xi^4}\frac{\cosh 2\left(1-\frac{|n|}{k}\right)\xi-1}{1-\frac{|n|}{k}}\right]$$

$$= i\varepsilon_{ijk}n_j\sigma^k \frac{1}{16\pi^2 k}\frac{\xi^2}{\sinh^3\xi}\left[\frac{2}{\xi^2}+\frac{\sinh 2\xi}{\xi^3}-\frac{2}{\xi^4}(\cosh 2\xi-1)\right.$$

$$+\frac{4}{\xi^2}N\left(1-\frac{N+1}{2k}\right)+\frac{1}{\xi^3}\frac{\cosh\left(2-\frac{1}{k}\right)\xi-\cosh(2s-1)\frac{\xi}{k}}{\sinh\frac{\xi}{k}}$$

$$\left.-\frac{4}{\xi^4}\int_0^\xi d\eta\,\frac{\cosh\left(2-\frac{1}{k}\right)\eta-\cosh(2s-1)\frac{\eta}{k}}{\sinh\frac{\eta}{k}}\right]. \tag{4.35}$$

After the substitution of this expression in eq. (4.19) the resulting integral over ξ cannot be expressed in elementary functions. But the $k\to\infty$ asymptotics can be found explicitly. Since the integral is infrared convergent, we may simply expand two last terms in (4.35) in powers of $1/k$, which gives, after integration:

$$k\frac{\Gamma_I^{(3)}}{\partial k}=\left(\frac{1}{3}+\frac{\pi^2}{9}\right)N\left(1-\frac{N+1}{2k}\right)+(3-2\ln 2\pi)k+\left(\frac{1}{6}+\frac{\pi^2}{18}\right)+O\left(\frac{1}{k}\right). \tag{4.36}$$

This completes the calculation of $\partial\Gamma_I/\partial k$. The result is given by (4.30) and (4.36); $\Gamma_I^{(2)}$ gives $O(1/k)$ contribution, as it is shown in Appendix. To obtain Γ_I one should remove the variation with respect to k. An integration of $\partial\Gamma_I^{(1)}/\partial k$ and of the last two terms in $\partial\Gamma_I^{(3)}/\partial k$ is trivial. As for the first term in $\partial\Gamma_I^{(3)}/\partial k$, it is convenient to divide the integration region in the integral intervals, i.e. to integrate first over $s=k\bmod 1$ from 0 to 1 at fixed $N\equiv k-s$ and then to sum over N:

$$\sum_{N<k}\int_0^1 ds\,\frac{N}{N+s}\left(1-\frac{N+1}{2}\frac{1}{N+s}\right)=\sum_{N<k}\left(\frac{1}{2}-\frac{1}{2N}+O\left(\frac{1}{N^2}\right)\right)$$

$$=\frac{1}{2}k-\frac{1}{2}\ln k+O\left(k^0\right).$$

Note that the term, proportional to $\ln k$, cancels with the result of the integration of the last one in (4.36). Adding up all contributions we obtain:

$$\Gamma_I=\frac{2}{3}k\ln k+\left(\frac{2}{3}\ln\frac{2\pi R}{\beta}+\frac{20}{9}+\frac{2\pi^2}{27}-2\ln 2-\frac{8}{3}\ln\pi+\frac{2\gamma}{3}\right)k+O\left(k^0\right). \tag{4.37}$$

5   Ultraviolet divergent part of determinant

In this section the surface term (3.22) is considered. It contains, since $\langle x|(-D^2)^{-2}|y\rangle\sim\frac{1}{16\pi^2}\ln(x-y)^2$, the logarithmic ultraviolet divergence of the form $-\frac{2}{3}k\ln\Lambda$. This is the conventional contribution of a scalar loop to the coupling constant renormalization. To calculate Γ_{II} completely we define

$$S^{ab}(\xi,\xi')=\int d^4\eta\,G^{ac}(\xi,\eta)G^{cb}(\eta,\xi'). \tag{5.1}$$

where the integration ranges over the whole Euclidean space and the rescaled variables (4.6) are used. The squared propagator, $(-D^2)^{-2}$, which enters eq. (3.22), can be obtained from S by the same procedure as the periodic Green function was obtained from the nonperiodic one in (4.10). But, since $S(\xi, \xi')$ depends only on the difference $\xi'_0 - \xi_0$, it is more convenient to Fourier transform in ξ_0. In such momentum–coordinate representation the asymptotics of the heat kernel for large Λ has the form

$$\mathrm{e}^{\,D^2/\Lambda^2} \sim \delta(\mathbf{x} - \mathbf{y})\,\mathrm{e}^{-\omega^2 \mu^2/\Lambda^2} \tag{5.2}$$

and, as

$$\left(\mu \frac{\partial A_0}{\partial \mu} F_{i0}\right)^{ab} = \frac{\mu}{R^2} n_i (n_a n_b - \delta_{ab}) + O\left(\mathrm{e}^{-\mu R}\right), \tag{5.3}$$

we rewrite eq. (3.22) as follows:

$$k \frac{\partial \Gamma_{II}}{\partial k} = \frac{1}{3} \oint_{S_R} d^2\Sigma \, \frac{1}{R^2} \sum_{n=-\infty}^{+\infty} \phi(\omega_n, \boldsymbol{\xi})\,\mathrm{e}^{-\omega_n^2 \mu^2/\Lambda^2}, \qquad \omega_n = \frac{n}{k}, \tag{5.4}$$

where

$$\phi(\omega, \boldsymbol{\xi}) = \int_{-\infty}^{+\infty} dt\ \mathrm{e}^{-i\omega t} (n_a n_b - \delta_{ab}) S^{ab}(0, \boldsymbol{\xi}; t, \boldsymbol{\xi}). \tag{5.5}$$

This function can be evaluated using the explicit form of the propagator (4.8). Note that the second term in eq. (4.8) is proportional to $n_a n_b$ and thus does not contribute to $\phi(\omega, \boldsymbol{\xi})$. The integration over η_0 in (5.1) and over t in (5.5) can be easily done, which yields:

$$
\begin{aligned}
\phi(\omega, \boldsymbol{\xi}) &= \int d^3\eta \, \frac{1}{64\pi^2 (\boldsymbol{\xi} - \boldsymbol{\eta})^2} (n_a n_b - \delta_{ab}) \Big(2\,\mathrm{tr}\,\big\langle v \big| \hat{Q}^a(\omega) \big| v \big\rangle \big\langle v \big| \hat{Q}^b(-\omega) \big| v \big\rangle \\
&\qquad - \mathrm{tr}\,\big\langle v \big| \hat{Q}^a(\omega) \big| v \big\rangle \ \mathrm{tr}\,\big\langle v \big| \hat{Q}^b(-\omega) \big| v \big\rangle \Big),
\end{aligned} \tag{5.6}
$$

where $|v\rangle \equiv |v(0, \boldsymbol{\eta})\rangle$ and

$$\big\langle z \big| \hat{Q}^a(\omega) \big| w \big\rangle = v(0, \boldsymbol{\xi}; z) \sigma^a v^\dagger(0, \boldsymbol{\xi}; w)\,\mathrm{e}^{-|z - w + \omega| \, |\boldsymbol{\xi} - \boldsymbol{\eta}|}. \tag{5.7}$$

The integral (5.6) must be evaluated for $\xi \to \infty$. In this limit only the domain of integration with $\eta \sim \xi$ gives essential contribution. In this region $v(0, \boldsymbol{\eta}; z)$ and $v(0, \boldsymbol{\xi}; z)$ are exponentially small, unless z is closed to $\pm 1/2$, when $v(0, \boldsymbol{\eta}; z) \sim \frac{1}{2}\sqrt{\eta}\,\mathrm{e}^{(1/2 \mp z)\eta}(1 \mp \sigma^i \nu_i)$ and $v(0, \boldsymbol{\xi}; z) \sim \frac{1}{2}\sqrt{\xi}\,\mathrm{e}^{(1/2 \mp z)\xi}(1 \mp \sigma^i n_i)$, where $\nu_i = \eta_i/\eta$ and $n_i = \xi_i/\xi$. Then, with exponential accuracy,

$$
\begin{aligned}
\big\langle v \big| \hat{Q}^a(\omega) \big| v \big\rangle &= \frac{1}{4}\xi\eta \sum_{p,q=\pm 1} \int_{-1/2} dz\,dw\ \mathrm{e}^{(1-z-w)(\xi+\eta) - |pz + qw + \omega| \, |\boldsymbol{\xi} - \boldsymbol{\eta}|} \\
&\qquad \times (1 + p\sigma^i \nu_i)(1 + p\sigma^j n_j)\sigma^a (1 - q\sigma^k n_k)(1 - q\sigma^l \nu_l).
\end{aligned} \tag{5.8}
$$

The substitution of this expression for $\big\langle v \big| \hat{Q}^a(\omega) \big| v \big\rangle$ in eq. (5.6) considerably simplifies the matrix algebra, which can be performed with the use of the identities

$$(1 + p\sigma^i n_i)\sigma^j n_j = \sigma^j n_j (1 + p\sigma^i n_i) = p(1 + p\sigma^i n_i),$$

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$$(1 + p\sigma^i n_i)(1 + q\sigma^j n_j) = 2\delta_{pq}(1 + p\sigma^i n_i).$$

After some calculations the integral (5.6) can be rewritten in the following compact form:

$$\phi(\omega, \boldsymbol{\xi}) = -\frac{1}{4\pi^2} \int d^3\eta\, \xi^2\eta^2 (1 + n_i\nu_i)^2 \left(M^2(\omega) + M^2(-\omega)\right), \tag{5.9}$$

where

$$M(\omega) = \int_{-1/2} dz\, dw\, e^{(1-z-w)(\xi+\eta)-|z+w-\omega||\boldsymbol{\xi}-\boldsymbol{\eta}|}. \tag{5.10}$$

It is convenient to change the integration variables in (5.9) to

$$x = \xi + \eta, \quad y = |\boldsymbol{\xi} - \boldsymbol{\eta}|, \tag{5.11}$$

then

$$\phi(\omega, \boldsymbol{\xi}) = -\frac{1}{8\pi\xi} \int_0^\infty \frac{dy}{y} \int_{\xi+|\xi-y|}^{2\xi+y} dx\, (x - \xi)(x^2 - y^2)^2 \left(M^2(\omega) + M^2(-\omega)\right). \tag{5.12}$$

Assuming that an upper bound of integration in (5.10) is equal to infinity, one finds:

$$M(\omega) = \begin{cases} \dfrac{e^{(\omega-1)y}}{(x+y)^2}, & \omega \le 1 \\[2ex] \dfrac{e^{-(\omega-1)y}}{(x-y)^2} - e^{-(\omega-1)x}\left[\dfrac{2(\omega-1)y}{x^2 - y^2} + \dfrac{4xy}{(x^2 - y^2)^2}\right], & \omega \ge 1. \end{cases} \tag{5.13}$$

When $|\omega| \ne 1$, the integrand in (5.12) is exponentially small for $y \sim \xi$, so one can restrict the integration region to $y \ll \xi$. Then the integral can be easily calculated and one obtains, up to $O(1/\xi)$ terms,

$$\phi(\omega, \boldsymbol{\xi}) = \begin{cases} -\dfrac{1}{4\pi(1 - \omega^2)}, & |\omega| < 1 \\[2ex] -\dfrac{|\omega|}{4\pi(\omega^2 - 1)}, & |\omega| > 1. \end{cases} \tag{5.14}$$

This expression is singular at $\omega = 1$. Remind that in the dimensionless units we use the size of the monopole is equal to unity. In fact, eq. (5.14) is not valid in the resonant case. The point is that, when $|\omega^2 - 1| \lesssim 1/\xi$, one can not restrict the integration region to $y \ll \xi$. In particular, $\phi(\pm 1, \boldsymbol{\xi})$ is finite of order ξ. The general expression for ϕ, valid also for resonant frequencies, is too cumbersome, but in what follows we shall need only the following integrals:

$$I_-(\triangle\omega, \boldsymbol{\xi}) = \int_{1-\triangle\omega}^1 d\omega\, \phi(\omega, \boldsymbol{\xi}), \quad I_+(\triangle\omega, \boldsymbol{\xi}) = \int_1^{1+\triangle\omega} d\omega\, \phi(\omega, \boldsymbol{\xi}). \tag{5.15}$$

For $1 \gg \triangle\omega \gg 1/\xi$ one gets:

$$I_-(\triangle\omega, \boldsymbol{\xi}) = -\frac{1}{8\pi}\left(\ln\xi\triangle\omega - \frac{5}{2} + \frac{\pi^2}{3} - 7\ln 2 + \gamma\right), \tag{5.16}$$

$$I_+(\triangle\omega, \boldsymbol{\xi}) = -\frac{1}{8\pi}\left(\ln\xi\triangle\omega + \frac{3}{2} + \ln 2 + \gamma\right). \tag{5.17}$$

Returning to the eq. (5.4) we find that, when the fractional part of k is not too close to 0 or to 1, say $N\triangle\omega < s < 1 - (N+1)\triangle\omega$, eq. (5.14) can be used for $\phi(\omega_n, \boldsymbol{\xi})$. On the other hand, when $s < N\triangle\omega$ or $s > 1 - (N+1)\triangle\omega$, the sum entering (5.4) can be approximated by $2\phi(\omega_N, \boldsymbol{\xi})$ or by $2\phi(\omega_{N+1}, \boldsymbol{\xi})$, respectively. By noting that

$$\int_0^{N\triangle\omega} \frac{ds}{k} \phi(\omega_N, \boldsymbol{\xi}) = I_-(\triangle\omega, 2\pi N T\mathbf{x}),$$

$$\int_{1-(N+1)\triangle\omega}^1 \frac{ds}{k} \phi(\omega_{N+1}, \boldsymbol{\xi}) = I_+(\triangle\omega, 2\pi(N+1)T\mathbf{x}),$$

we find:

$$
\begin{aligned}
\Gamma_{II} &= \frac{1}{3}\int^k dk' \frac{4\pi}{k'} \sum_{n=-\infty}^{+\infty} \phi(\omega_n, \boldsymbol{\xi})\, e^{-\omega_n^2 \mu^2/\Lambda^2} \\
&= \frac{1}{3}\sum_{N<k} \{8\pi I_-(\triangle\omega, 2\pi N T\mathbf{x}) + 8\pi I_+(\triangle\omega, 2\pi(N+1)T\mathbf{x}) \\
&\quad - \int_{N\triangle\omega}^{1-(N+1)\triangle\omega} ds \left[2\ln\frac{\Lambda}{2\pi T} + 2\psi(2N+s+1) - 2\psi(s) \right. \\
&\quad \left. -2\psi(N+1) - \pi\cot\pi(N+s) - \gamma \right] \} \\
&= -\frac{1}{3}k\left(2\ln R\Lambda - 1 + \frac{\pi^2}{3} - 4\ln 2 + \gamma \right) + O\left(k^0\right).
\end{aligned}
\tag{5.18}
$$

Collecting together all contributions, given by eqs. (3.24), (4.37) and (5.18), we obtain the final result, quoted in sec. 2:

$$\Gamma = \frac{2}{3}k\ln\frac{T}{\Lambda} + \frac{2}{3}k\ln k + \left(\frac{20}{9} - \frac{\pi^2}{27} - 2\ln\pi + \frac{\gamma}{3} \right) k + O\left(k^0\right). \tag{5.19}$$

6 Conclusions

Our results show that, despite the Coulomb nature of the monopole field, the one–loop corrections to the action are free from the infrared divergences. Thus the contribution of the BPS monopoles can, in principle, be calculated by semiclassical methods. To perform this calculation completely it is important to consider zero modes in the monopole background and related nonstatic deformations of the classical solution. The number of the zero modes grows with the decrease of the monopole size, which makes the problem of the calculation of the monopole density more complicated, than that for the instantons.

It is worth mentioning that generalization to $SU(N_c)$ with $N_c > 2$ and introduction of massless fermions also should not be literally analogous to that in the instanton calculations, because of a difference in the structure of gauge zero modes and the fact that the topological charge of the BPS monopole, and hence the number of fermion zero modes, is related to the monopole size. In particular, the form of the 't Hooft interaction [17], induced by the BPS monopoles, may be not the same as for the instantons.

Acknowledgments

Appendix A   Calculation of $\Gamma_I^{(2)}$

In this appendix we calculate $\Gamma_I^{(2)}$, the variation of which is given by (4.19) with $J^{(2)}$ defined in (4.32). First, we note that

$$
\begin{aligned}
\left\langle v \left| \left[\partial_\mu \hat{P}, \, \mathrm{e}^{2\pi i k n z} \right] \right| v \right\rangle &= \left\langle \partial_\mu v \left| \mathrm{e}^{2\pi i k n z} \right| v \right\rangle - \left\langle v \left| \mathrm{e}^{2\pi i k n z} \right| \partial_\mu v \right\rangle \\
&\quad + \left\{ A_\mu, \left\langle v \left| \mathrm{e}^{2\pi i k n z} \right| v \right\rangle \right\}.
\end{aligned} \tag{A.1}
$$

We begin with the term, containing $J_0^{(2)}$. Define

$$
E_n \equiv \left\langle v \left| \mathrm{e}^{2\pi i k n z} \right| v \right\rangle = C_n - i n_i \sigma^i D_n, \tag{A.2}
$$

$$
C_n = \frac{\xi^2 \cos \pi k n + \xi \coth \xi \, \pi k n \sin \pi k n}{\xi^2 + \pi^2 k^2 n^2}, \tag{A.3}
$$

$$
D_n = \frac{\xi^2 \coth \xi \, \sin \pi k n - \xi \pi k n \cos \pi k n}{\xi^2 + \pi^2 k^2 n^2}, \tag{A.4}
$$

then, since $\left| \partial_0 v \right\rangle = i z \left| v \right\rangle$,

$$
\left\langle v \left| \left[\partial_0 \hat{P}, \, \mathrm{e}^{2\pi i k n z} \right] \right| v \right\rangle = -\frac{1}{\pi n} \frac{\partial E_n}{\partial k} + \left\{ A_0, E_n \right\}. \tag{A.5}
$$

Substituting this expression into (4.32) one gets:

$$
\begin{aligned}
J_0^{(2)} &= \frac{1}{8\pi^2} \sum_{n \neq 0} \left[\frac{1}{\pi^3 k^3 n^3} \left(1 - \frac{1}{2} k \frac{\partial}{\partial k} \right) \left(E_n \, \mathrm{tr} \, E_{-n} - E_{-n} \, \mathrm{tr} \, E_n \right) \right. \\
&\quad + \frac{1}{2\pi^2 k^2 n^2} \left\{ A_0, E_n \, \mathrm{tr} \, E_{-n} + E_{-n} \, \mathrm{tr} \, E_n \right\} \\
&\quad \left. + \frac{1}{\pi^2 k^2 n^2} E_{-n} \, \mathrm{tr} \, \left(A_0 E_n \right) - \frac{1}{\pi^2 k^2 n^2} E_n \, \mathrm{tr} \, \left(A_0 E_{-n} \right) \right].
\end{aligned} \tag{A.6}
$$

Using (A.2) and taking into account that

$$
\frac{1}{k^3} \left(1 - \frac{1}{2} k \frac{\partial}{\partial k} \right) = -\frac{1}{2} \frac{\partial}{\partial k} \frac{1}{k^2},
$$

we find:

$$
\begin{aligned}
J_0^{(2)} &= \frac{1}{2\pi^2} \left[i n_i \sigma^i \frac{1}{2} \frac{\partial}{\partial k} \left(\frac{1}{\pi^3 k^2} \sum_{n \neq 0} \frac{C_n D_n}{n^3} \right) + A_0 \frac{1}{\pi^2 k^2} \sum_{n \neq 0} \frac{C_n^2 - D_n^2}{n^2} \right] \\
&= i n_i \sigma^i \frac{1}{4\pi^2} \left[\frac{\partial P}{\partial k} - \left(\coth \xi - \frac{1}{\xi} \right) Q \right],
\end{aligned} \tag{A.7}
$$

where

$$P = \frac{1}{\pi^3 k^2} \sum_{n \neq 0} \frac{C_n D_n}{n^3} = \frac{1}{2\pi^3 k^2} \left[\xi^4 \coth \xi \sum_{n \neq 0} \frac{\sin 2\pi k n}{n^3 \left(\xi^2 + \pi^2 k^2 n^2 \right)^2} \right.$$

$$-\pi^2 k^2 \xi^2 \coth \xi \sum_{n \neq 0} \frac{\sin 2\pi k n}{n \left(\xi^2 + \pi^2 k^2 n^2 \right)^2}$$

$$-\pi k \frac{\xi^3 \cosh 2\xi}{\sinh^2 \xi} \sum_{n \neq 0} \frac{\cos 2\pi k n}{n^2 \left(\xi^2 + \pi^2 k^2 n^2 \right)^2}$$

$$\left. +\pi k \frac{\xi^3}{\sinh^2 \xi} \sum_{n \neq 0} \frac{1}{n^2 \left(\xi^2 + \pi^2 k^2 n^2 \right)^2} \right], \qquad \text{(A.8)}$$

$$Q = \frac{1}{\pi^2 k^2} \sum_{n \neq 0} \frac{C_n^2 - D_n^2}{n^2} = \frac{1}{2\pi^2 k^2} \left[\frac{\xi^4 \cosh 2\xi}{\sinh^2 \xi} \sum_{n \neq 0} \frac{\cos 2\pi k n}{n^2 \left(\xi^2 + \pi^2 k^2 n^2 \right)^2} \right.$$

$$-\pi^2 k^2 \frac{\xi^2 \cosh 2\xi}{\sinh^2 \xi} \sum_{n \neq 0} \frac{\cos 2\pi k n}{\left(\xi^2 + \pi^2 k^2 n^2 \right)^2}$$

$$+4\pi k \xi^3 \coth \xi \sum_{n \neq 0} \frac{\sin 2\pi k n}{n \left(\xi^2 + \pi^2 k^2 n^2 \right)^2}$$

$$\left. +\frac{\xi^2}{\sinh^2 \xi} \sum_{n \neq 0} \frac{\pi^2 k^2 n^2 - \xi^2}{n^2 \left(\xi^2 + \pi^2 k^2 n^2 \right)^2} \right]. \qquad \text{(A.9)}$$

The sums entering (A.8) and (A.9) can be calculated as $t \to 0$ or $t \to \infty$ limits of the following equality:

$$\sum_{n \neq 0} \frac{\mathrm{e}^{2\pi i k n}}{(n + it) \left(\xi^2 + \pi^2 k^2 n^2 \right)^2} = \frac{i}{\xi^4 t} - \frac{i\pi}{\xi^4} \frac{\mathrm{e}^{(2s-1)\pi t}}{\left(1 - \frac{\pi^2 k^2 t^2}{\xi^2} \right)^2 \sinh \pi t}$$

$$+\frac{i\pi}{2\xi^4} \frac{2 \sinh(2s-1) \frac{\xi}{k} + \frac{\pi k t}{\xi} \left(3 - \frac{\pi^2 k^2 t^2}{\xi^2} \right) \cosh(2s-1) \frac{\xi}{k}}{\left(1 - \frac{\pi^2 k^2 t^2}{\xi^2} \right)^2 \sinh \frac{\xi}{k}}$$

$$-\frac{i\pi s}{k\xi^3} \frac{\cosh(2s-1) \frac{\xi}{k} + \frac{\pi k t}{\xi} \sinh(2s-1) \frac{\xi}{k}}{\left(1 - \frac{\pi^2 k^2 t^2}{\xi^2} \right) \sinh \frac{\xi}{k}} + \frac{i\pi}{2k\xi^3} \frac{\sinh 2s \frac{\xi}{k} + \frac{\pi k t}{\xi} \cosh 2s \frac{\xi}{k}}{\left(1 - \frac{\pi^2 k^2 t^2}{\xi^2} \right) \sinh^2 \frac{\xi}{k}}, \text{ (A.10)}$$

which can be obtained by Poisson resummation. Here s is the fractional part of k. The result of the calculations reads:

$$P = \frac{1}{3k^2} \left(2s^3 - 3s^2 + s \right) \coth \xi - \frac{1}{6k} \left(6s^2 - 6s + 1 \right) \frac{\cosh 2\xi}{\xi \sinh^2 \xi}$$

$$+\frac{3}{2}(2s-1) \frac{\coth \xi}{\xi^2} - k \frac{\cosh 2\xi}{\xi^3 \sinh^2 \xi} + \frac{1}{4k} \frac{\cosh 2\xi}{\xi \sinh^2 \xi} \frac{\cosh 2s \frac{\xi}{k}}{\sinh^2 \frac{\xi}{k}}$$

$$-\frac{1}{2k} \frac{\coth \xi}{\xi} \frac{\sinh 2s \frac{\xi}{k}}{\sinh^2 \frac{\xi}{k}} + \left(\frac{s}{k} \frac{\coth \xi}{\xi} + \frac{3}{4} \frac{\cosh 2\xi}{\xi^2 \sinh^2 \xi} \right) \frac{\cosh(2s-1) \frac{\xi}{k}}{\sinh \frac{\xi}{k}}$$

$$-\left(\frac{s}{2k} \frac{\cosh 2\xi}{\xi \sinh^2 \xi} + \frac{3}{2} \frac{\coth \xi}{\xi^2} \right) \frac{\sinh(2s-1) \frac{\xi}{k}}{\sinh \frac{\xi}{k}}$$

$$+\frac{1}{2\pi^2 k}\frac{\xi^3}{\sinh^2\xi}\sum_{n\neq 0}\frac{1}{n^2\left(\xi^2+\pi^2 k^2 n^2\right)^2}, \tag{A.11}$$

$$
\begin{aligned}
Q &= \frac{1}{6k^2}\left(6s^2-6s+1\right)\frac{\cosh 2\xi}{\sinh^2\xi}-\frac{2}{k}(2s-1)\frac{\coth\xi}{\xi}+\frac{3}{2}\frac{\cosh 2\xi}{\xi^2\sinh^2\xi}\\
&\quad -\frac{1}{2k^2}\frac{\cosh 2\xi}{\sinh^2\xi}\frac{\cosh 2s\frac{\xi}{k}}{\sinh^2\frac{\xi}{k}}+\frac{1}{k^2}\coth\xi\frac{\sinh 2s\frac{\xi}{k}}{\sinh^2\frac{\xi}{k}}\\
&\quad -\left(\frac{2s}{k^2}\coth\xi+\frac{1}{k}\frac{\cosh 2\xi}{\xi\sinh^2\xi}\right)\frac{\cosh(2s-1)\frac{\xi}{k}}{\sinh\frac{\xi}{k}}\\
&\quad +\left(\frac{s}{k^2}\frac{\cosh 2\xi}{\sinh^2\xi}+\frac{2}{k}\frac{\coth\xi}{\xi}\right)\frac{\sinh(2s-1)\frac{\xi}{k}}{\sinh\frac{\xi}{k}}\\
&\quad +\frac{\xi^2}{2\sinh^2\xi}\sum_{n\neq 0}\frac{\pi^2 k^2 n^2-\xi^2}{\pi^2 k^2 n^2\left(\xi^2+\pi^2 k^2 n^2\right)^2}.
\end{aligned}
\tag{A.12}
$$

The substitution of (A.7) into (4.19) gives

$$
\begin{aligned}
k\frac{\partial\Gamma^{(2)}_{I,t}}{\partial k} &= 2k\left[\frac{\partial}{\partial k}\int_0^{\lambda k}d\xi\,\xi^2\left(\coth\xi-\frac{\xi}{\sinh^2\xi}\right)P-\lambda^3 k^2 P(\lambda k)\right.\\
&\quad \left. -\int_0^{\lambda k}d\xi\,\xi^2\left(\coth\xi-\frac{1}{\xi}\right)\left(\coth\xi-\frac{\xi}{\sinh^2\xi}\right)Q\right].
\end{aligned}
\tag{A.13}
$$

The leading infrared divergence comes from the first terms in P and Q and is easily calculable:

$$
\begin{aligned}
k\frac{\Gamma^{(2)}_{I,t}}{\partial k} &\sim 2k\left\{\frac{\partial}{\partial k}\left[\frac{1}{3}\lambda^3 k\frac{1}{3}\left(2s^3-3s^2+s\right)\right]-\lambda^3\frac{1}{3}\left(2s^3-3s^2+s\right)\right.\\
&\quad \left. -\frac{2}{3}\lambda^3 k\frac{1}{6}\left(6s^2-6s+1\right)\right\}=-\frac{4}{9}k\lambda^3\left(2s^3-3s^2+s\right).
\end{aligned}
$$

Integration over k gives

$$\ln\det\left(-D^2\right)\sim\frac{2}{9}\lambda^3 s^2(1-s)^2, \tag{A.14}$$

the result, quoted in Sec. 4.

For integral k (A.14) turns to zero. One may expect that other power–like divergences also cancel. We shall see that this is really the case, moreover, the logarithmic divergence vanishes as well, and the resulting contribution to the determinant is finite and is of order $1/k$.

After integration over ξ in (A.13) one should remove the variation with respect to k. This procedure is trivial for the first term and, as we are interested in the value of $\ln\det\left(-D^2\right)$ for integral k, we can put $s=0$ in the integrand. The second and the third terms should be integrated over k directly. We find:

$$\int_0^{\lambda k}d\xi\,\xi^2\left(\coth\xi-\frac{\xi}{\sinh^2\xi}\right)P\bigg|_{s=0}=-\frac{1}{6}\lambda^2 k-\frac{3}{2}\left(\lambda k-\frac{1}{2}\right)$$

$$-2k\left(\ln\frac{\lambda k}{\pi}+\frac{2}{3}+\gamma+2\zeta'(-2)\right)+\frac{1}{2}k\left(\ln\frac{k}{2\pi}+\frac{5}{3}+\gamma+2\zeta'(-2)\right)$$

$$+\frac{3}{2}k\left(\ln\frac{k}{2\pi}+\lambda+\frac{2}{3}+\gamma+2\zeta'(-2)\right)-\frac{3}{2}\left(-\lambda k+\frac{1}{2}\right)+O\left(\frac{1}{k}\right) \tag{A.15}$$

$$\int_0^{\lambda k}d\xi\,\xi^2\left(\coth\xi-\frac{1}{\xi}\right)\left(\coth\xi-\frac{\xi}{\sinh^2\xi}\right)Q$$

$$=\frac{1}{6k^2}\left(6s^2-6s+1\right)\left(\frac{2}{3}\lambda^3k^3-\lambda^2k^2\right)$$

$$-\frac{2}{k}(2s-1)\left(\frac{1}{2}\lambda^2k^2-\lambda k+\frac{1}{3}+\frac{\pi^2}{36}\right)$$

$$+3\left(\lambda k-\ln\frac{\lambda k}{\pi}-\frac{17}{12}-\gamma-2\zeta'(-2)\right)$$

$$-\frac{1}{2k^2}\left[k^3\left(F'_-(s)+\frac{1}{2}sF''_-(s)\right)\right.$$

$$\left.+k^2\left(F_+(s)+sF'_+(s)-2\ln\frac{k}{2\pi}-\frac{29}{6}-4\zeta'(-2)\right)\right]$$

$$-\frac{1}{k^2}\left[\frac{1}{2}k^3\left(F'_+(s)+\frac{1}{2}sF''_+(s)\right)+\frac{1}{2}k^2\left(F_-(s)+sF'_-(s)\right)-2sk\left(\frac{1}{3}+\frac{\pi^2}{36}\right)\right]$$

$$-\frac{2s}{k^2}\left[k^3F''_1(\Delta)-k^2F'_2(\Delta)+k\left(\frac{1}{3}+\frac{\pi^2}{36}\right)\right]$$

$$-\frac{2}{k}\left[k^2F'_2(\Delta)-k\left(\ln\frac{k}{2\pi}+F_1(\Delta)+\frac{17}{12}+2\zeta'(-2)\right)\right]$$

$$+\frac{2s}{k^2}\text{sign}(2s-1)\left(-k^3F''_2(\Delta)+k^2F'_1(\Delta)\right)$$

$$+\frac{2}{k}\left[\text{sign}(2s-1)\left(-k^2F'_1(\Delta)+kF_2(\Delta)\right)+(2s-1)\left(\frac{1}{3}+\frac{\pi^2}{36}\right)\right]$$

$$+O\left(\frac{1}{k^2}\right), \tag{A.16}$$

where

$$\Delta(s)=1-|2s-1|, \tag{A.17}$$

$$F_1(\Delta)=\frac{1-e^{-\lambda\Delta}}{\Delta}-\frac{1}{2}\left(\psi\left(1+\frac{\Delta}{2}\right)+\psi\left(1-\frac{\Delta}{2}\right)\right), \tag{A.18}$$

$$F_2(\Delta)=\frac{e^{-\lambda\Delta}}{\Delta}-\frac{\pi}{2}\cot\frac{\pi\Delta}{2}, \tag{A.19}$$

$$F_\pm(s)=\psi(s)\pm\psi(-s). \tag{A.20}$$

For the second term in (A.13) we have:

$$\lambda^3k^2P(\lambda k)=\frac{1}{3}\left(2s^3-3s^2+s\right)\lambda^3-\frac{1}{3}\left(6s^2-6s+1\right)\lambda^2+\frac{3}{2}(2s-1)\lambda$$

$$-2+(1-\text{sign}(2s-1))\left(\lambda^2s+\frac{3}{2}\lambda\right)e^{-\lambda\Delta}, \tag{A.21}$$

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but it gives zero contribution:

$$\int_0^1 ds\, \lambda^3 k^2 P(\lambda k) = 0. \tag{A.22}$$

Let us consider the contribution of (A.16). Some of the terms in this expression do not depend on s explicitly and can be integrated over k directly; others should be first integrated over s from 0 to 1 at fixed N and then summed over N. Thus we obtain:

$$\begin{aligned}
\int^k dk' \int_0^{\lambda k'} d\xi\, \xi^2 \left(\coth\xi - \frac{1}{\xi}\right)\left(\coth\xi - \frac{\xi}{\sinh^2\xi}\right)Q \\
= \int^k dk' \left(3\lambda k' - 3\ln 2\lambda + 1 - 3\gamma + O\left(\frac{1}{k'^2}\right)\right) \\
+ \sum_{N<k}\left(-\frac{1}{6}\lambda^2 - 3\lambda N + \ln 2\lambda - \frac{1}{2} + 3\gamma + O\left(\frac{1}{N^2}\right)\right) \\
= \left(-\frac{1}{6}\lambda^2 + \frac{3}{2}\lambda - 2\ln 2\lambda + \frac{1}{2}\right)k + O\left(\frac{1}{k}\right) \tag{A.23}
\end{aligned}$$

The final result is given by a difference of (A.15) and (A.23) and is equal to zero up to the terms of order $1/k$.

For the spatial components of the current we find

$$\left\langle v\left|\left[\partial_i \hat{P},\, \mathrm{e}^{2\pi iknz}\right]\right|v\right\rangle = i\varepsilon_{ijk}n_j\sigma^k \frac{1}{\sinh\xi}\left(C_n - \frac{\sin\pi nk}{\pi nk}\right) \tag{A.24}$$

and

$$J_i^{(2)} = i\varepsilon_{ijk}n_j\sigma^k \frac{1}{4\pi^2}\frac{1}{\sinh\xi}\sum_{n\neq 0}\left(\frac{C_n^2}{\pi^2 k^2 n^2} - \frac{C_n\sin\pi kn}{\pi^3 k^3 n^3}\right). \tag{A.25}$$

The substitution of this expression in (4.19) leads to the convergent integral, so the limit $k\to\infty$ and integration over ξ can be interchanged. Since $J_i^{(2)}$ is, at least, $o(1/k^3)$, it's contribution is negligible at large k. Thus we conclude that $\Gamma_I^{(2)} = O(1/k)$.

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